

Measuring a Light Neutralino Mass at the ILC: Testing the MSSM Neutralino Cold Dark Matter Model

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The LEP experiments give a lower bound on the neutralino mass of about 46 GeV which, however, relies on a supersymmetric grand unification relation. Dropping this assumption, the experimental lower bound on the neutralino mass vanishes completely. Recent analyses suggest, however, that in the minimal supersymmetric standard model (MSSM), a light neutralino dark matter candidate has a lower bound on its mass of about 7 GeV. In light of this, we investigate the mass sensitivity at the ILC for very light neutralinos. We study slepton pair production, followed by the decay of the sleptons to a lepton and the lightest neutralino. We find that the mass measurement accuracy for a few-GeV neutralino is around 2 GeV, or even less if the relevant slepton is sufficiently light. We thus conclude that the ILC can help verify or falsify the MSSM neutralino cold dark matter model even for very light neutralinos.

I. INTRODUCTION

The supersymmetric Standard Model (SSM) [1, 2] is a well motivated extension of the Standard Model of particle physics which solves the hierarchy problem between the weak scale and the Planck scale [3, 4]. In order to guarantee a stable proton, usually a discrete symmetry beyond the SM gauge symmetries is imposed which prohibits the baryon- and lepton-number violating terms in the superpotential: R-parity [5], proton hexality [6], or a \mathbf{Z}_4 R-symmetry [7]. This is then called the minimal supersymmetric Standard Model (MSSM). The discrete symmetry furthermore guarantees that the lightest supersymmetric particle (LSP) is stable and thus a dark matter candidate [8–11]. If it is to constitute the entire dark matter in the universe it must be electrically and color neutral [8]. Here we focus on the lightest neutralino χ_1^0 as the LSP. In order to avoid overclosure of the universe, the neutralino must be either very light $M_{\chi_1^0} < \mathcal{O}(1 \text{ eV})$ (Cowsik–McLelland bound) [12, 13] or heavy $M_{\chi_1^0} > \mathcal{O}(10 \text{ GeV})$ (Lee–Weinberg bound) [14]. We shall make this latter lower bound more precise [15–21]. In this paper we are interested in how a neutralino mass close to the Lee–Weinberg bound could be measured at the ILC. This is potentially a stringent test of a MSSM light dark matter model.

Within the MSSM the spin-1/2 superpartners of the hypercharge B boson, the neutral $SU(2)$ W boson and the two CP-even neutral Higgs bosons mix after electroweak symmetry breaking. The resulting four mass eigenstates are the neutralinos and are denoted χ_i^0 , $i = 1, \dots, 4$. The masses are ordered $M_{\chi_1^0} < \dots < M_{\chi_4^0}$. If produced at colliders, the lightest neutralino behaves like a heavy stable neutrino and escapes detection. The spin-1/2 superpartners of the charged $SU(2)$ W boson and of the charged Higgs boson also mix after electroweak symmetry breaking. The resulting mass eigenstates are the charginos and denoted $\chi_{i=1,2}^\pm$, with ordered masses. See Appendix A for details.

The current Particle Data Group (PDG) mass bound from LEP on the lightest neutralino is [22–25]

$$M_{\chi_1^0} > 46 \text{ GeV}. \quad (1)$$

This bound is obtained by searching for charginos and thus setting a bound on the $SU(2)$ gaugino mass M_2 and the Higgs mixing parameter μ . Using the supersymmetric grand unified theory (SUSY GUT) relation between M_2 and the $U(1)_Y$ gaugino mass term M_1

$$M_1 = \frac{5}{3} \tan^2 \theta_w M_2, \quad (2)$$

the chargino search can be translated into a bound on M_1 . The neutralino mass matrix is computed for all allowed values of the supersymmetric parameters, taking into account Eq. (2), as well as the lower bound on the ratio of the Higgs vacuum expectation values, $\tan \beta \gtrsim 2$, from the LEP Higgs search [25]. Then one obtains as the lowest possible neutralino mass the bound in Eq. (1).

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If, however, the assumption Eq. (2) is dropped, there is no lower laboratory or astrophysical bound on the neutralino mass [13, 26–32]. Even a massless neutralino is allowed. This is now included in the PDG as a comment [33]. There is however a cosmological bound which we now discuss.

Relaxing the SUSY GUT assumption in Eq. (2), it is possible to derive the Lee–Weinberg lower limit on the mass of the neutralino LSP, $M_{\chi_1^0}^{\min}$, in the MSSM with real parameters. It was first determined for large pseudoscalar Higgs masses [15, 16], obtaining $M_{\chi_1^0}^{\min} = \mathcal{O}(15 \text{ GeV})$. It was subsequently realized however [17, 18], that a region of parameter space exists with a low pseudoscalar Higgs mass and high $\tan\beta$, in which the neutralino lower mass limit reaches $M_{\chi_1^0}^{\min} \approx 6 \text{ GeV}$. This is due to an enhancement in the neutralino annihilation cross section from annihilation to b-quarks via Higgs bosons, which keeps the predicted relic density below the observed limits. This was confirmed in Ref. [34]. There it was furthermore noted that this area of parameter space would be testable at the Tevatron, for example, with the Higgs search results in which the Higgs is produced in association with a b-quark, as well as via the $B_s \rightarrow \mu^+ \mu^-$ limit. See also the more recent work in Ref. [35–38]. In a very recent paper [39], the authors argue that these constraints have a relatively minor impact on the light neutralino parameter space of the MSSM [83], and that the lower bound is

$$M_{\chi_1^0}^{\min} \approx 7 - 8 \text{ GeV}. \quad (3)$$

Recently there has also been an increased interest in light dark matter candidates with a mass of order 5 GeV due to the DAMA/LIBRA [40] and CoGeNT [41] direct search results. Ref. [42] suggests that the required scattering cross sections in the detectors cannot be obtained within the MSSM, though in Ref. [39] it is claimed that the current constraints do in fact allow sufficiently high cross sections for these experimental hints to be explained by an MSSM neutralino. In the NMSSM, it is similarly claimed in Ref. [43, 44] that high cross sections cannot be obtained, though Refs. [45, 46] identify regions of parameter space in this model in which the presence of a light singlet Higgs can lead to large enough cross sections. The authors of Refs. [47] argue that solutions also exist for an extended NMSSM. The experimental results in Refs. [40, 41] have lead to a flourish of alternative schemes, e.g. [48–58]. We note, however, that these models are severely constrained by the CDMS-II [59, 60] and XENON10 [61] and XENON100 data [62, 63].

Given these considerations it is thus of great interest how well the neutralino mass can be determined in the low-mass region. It is the goal of this paper to test the sensitivity of the mass measurement at the ILC. There is the possibility that a neutralino mass will be measured which is too small to be reconciled with the observed relic abundance, if the real MSSM and standard cosmology are assumed. Such a measurement would be striking

evidence for non-minimal supersymmetric models and/or non-standard cosmologies.

The paper is organized as follows. In section II, we provide an overview of some of the methods that have been suggested to measure the lightest neutralino mass at the ILC. We then focus, in Section III, on the neutralino mass measurement that can be done using slepton pair production, and make a first estimate of the accuracy of this method for a very light neutralino. In Section IV, we describe a simulation of slepton pair production at the ILC, and use it to make a better determination of the accuracy attainable for the neutralino mass measurement. We summarize and conclude in Section V.

II. NEUTRALINO MASS MEASUREMENTS

Several methods have been suggested in the literature to measure the mass of the lightest neutralino at the ILC [64–68]. Throughout, the authors have focused on a neutralino heavier than the LEP bound in Eq. (1). For example, the widely studied SPS1a point (without a slope) [69] has $M_{\chi_1^0} = 97.1 \text{ GeV}$. The most straightforward method involves considering slepton pair production, followed by the decay of each slepton to the lightest neutralino and a charged lepton

$$e^+ e^- \rightarrow \tilde{\ell}^- \tilde{\ell}^+ \rightarrow \ell^- \ell^+ + 2\chi_1^0, \quad \ell = e, \mu. \quad (4)$$

Here the (s)leptons are restricted to the first two generations. The measurement via the third generation (s)tau is diluted by the additional decay to the neutrino(s). Measuring the energies of the final state leptons, one can extract information on the neutralino and slepton masses. The typical relative precision achieved is in the per mille range [64, 66–68]. We go beyond this work and discuss this method in detail for a very light neutralino.

A second method in the literature is based on the pair production of the second lightest neutralinos. This is followed by the decay of each neutralino via a (virtual) slepton to a charged lepton pair and the lightest neutralino,

$$e^+ e^- \rightarrow \chi_2^0 \chi_2^0 \rightarrow (\chi_1^0 \ell_1^+ \ell_1^-) (\chi_1^0 \ell_2^+ \ell_2^-). \quad (5)$$

where each χ_2^0 decays independently and thus ℓ_1 need not equal ℓ_2 . In fact, the case $\ell_1 \neq \ell_2$ reduces the combinatorial uncertainty. In Refs. [70, 71] the authors then propose to measure the di-lepton invariant mass and the di-lepton energy and to use these to measure the two lightest neutralino masses.

Of necessity these methods also always involve other supersymmetric particles and their masses. For example, the first method relies on the production of sleptons. The second method relies on the production of the second lightest neutralino and then its decay to an intermediate slepton. Thus both of these methods can be improved by measuring the corresponding supersymmetric masses

directly. For example the slepton mass can be well determined by an energy scan over the slepton mass threshold [70]. Similarly a scan over the production threshold energy of the process given in Eq. (5) gives a tight constraint on the mass $M_{\chi_2^0}$ [70].

III. SLEPTON PAIR PRODUCTION AND THE NEUTRALINO MASS

In this section, we study the measurement of the lightest neutralino mass using slepton pair production at the ILC, as shown in Eq. (4). The slepton decay to a lepton and the lightest neutralino is a two-body decay. Therefore in the slepton rest-frame the lepton energy is completely fixed by the slepton and neutralino mass. Ignoring initial and final state radiation (ISR and FSR), beamstrahlung, and detector effects for the moment, the slepton energy is then just the beam energy. Thus the lepton's lab-frame energy E_ℓ is fully determined by the angle θ_0 , with which the slepton emits the lepton in the slepton rest-frame. The angle is measured with respect to the slepton lab momentum direction. We then have for the lepton energy

$$E_\ell = \frac{\sqrt{s}}{4} \left(1 - \frac{M_{\chi_1^0}^2}{M_{\tilde{\ell}}^2} \right) (1 + \beta \cos \theta_0). \quad (6)$$

Here $\beta = \sqrt{1 - 4m_\ell^2/s}$ is the slepton velocity in the lab frame, $\sqrt{s}/2$ is the beam energy, and $M_{\tilde{\ell}}$ denotes the slepton mass. The event distribution of E_ℓ is flat between its maximum E_+ , when $\cos \theta_0 = 1$, and its minimum E_- , when $\cos \theta_0 = -1$. The equations for E_+ and E_- can be inverted to find the slepton and neutralino masses squared in terms of these endpoints,

$$M_{\tilde{\ell}}^2 = s \frac{E_+ E_-}{(E_+ + E_-)^2}, \quad (7)$$

and

$$M_{\chi_1^0}^2 = M_{\tilde{\ell}}^2 \left(1 - \frac{E_+ + E_-}{\sqrt{s}/2} \right). \quad (8)$$

We have listed the squared formulæ for later use. Taking the positive square root we then obtain for the masses

$$M_{\tilde{\ell}} = \sqrt{s} \frac{\sqrt{E_+ E_-}}{E_+ + E_-}, \quad (9)$$

and

$$M_{\chi_1^0} = M_{\tilde{\ell}} \sqrt{1 - \frac{E_+ + E_-}{\sqrt{s}/2}}. \quad (10)$$

The sensitivity of the neutralino mass measurement thus depends on the accuracy with which E_\pm can be measured.

Looking at Eq. (6), it is clear that E_\pm only have a weak dependence on $M_{\chi_1^0}$ for $M_{\chi_1^0} \ll M_{\tilde{\ell}}$. Thus we would

expect the accuracy of the neutralino mass measurement to deteriorate for sufficiently small neutralino masses. We set out to quantify this below.

Limited statistics and detector and beam effects introduce uncertainty into the endpoint determination. Nonetheless, for typical slepton and heavy neutralino masses, the endpoints and the masses can be determined to sub-GeV accuracy [67]. For very light neutralinos, however, even small errors in the endpoint measurements can lead to a large fractional error in the neutralino mass determination.

Before studying this issue with a simulation, we can estimate the mass determination accuracy for a light neutralino by combining the quoted accuracy from an experimental study by Martyn [67] with a simple error analysis. Assuming that E_+ and E_- are independent random variables, then from Eq. (10) we can derive

$$\frac{\delta M_{\chi_1^0}}{\delta E_\pm} = \frac{\delta M_{\tilde{\ell}}}{\delta E_\pm} \frac{M_{\chi_1^0}}{M_{\tilde{\ell}}} - \frac{M_{\tilde{\ell}}^2}{M_{\chi_1^0} \sqrt{s}}. \quad (11)$$

For light neutralinos, the first term in Eq. (11) is negligible. The second term dominates and is identical for E_+ and E_- , so we can write

$$\delta M_{\chi_1^0} \simeq \frac{M_{\tilde{\ell}}^2}{M_{\chi_1^0} \sqrt{s}} \sqrt{\delta E_+^2 + \delta E_-^2}. \quad (12)$$

In the simulation we describe below, we consider SUSY scenarios with $M_{\tilde{e}_R} = 100$ and 200 GeV and varying $M_{\chi_1^0}$, a center-of-mass energy $\sqrt{s} = 500$ GeV and an integrated luminosity $\mathcal{L} = 250 \text{ fb}^{-1}$. For illustration, we here assume these experimental parameters as well as a 2 GeV neutralino mass and 100 GeV selectron mass. Thus the factor in front of the square root in Eq. (12) is 10.

In Ref. [67], the error on the endpoint determinations is given as $\delta E_+ = 0.11$ GeV and $\delta E_- = 0.02$ GeV for a scenario with $M_{\chi_1^0} = 93$ GeV, $M_{\tilde{\ell}} = 143$ GeV, $\mathcal{L} = 200 \text{ fb}^{-1}$, and $\sqrt{s} = 400$ GeV. In this scenario, because the neutralino is heavy the two terms in Eq. (11) are comparable, so we cannot use Eq. (12). Using Eq. (11) instead, we obtain $\delta M_{\chi_1^0} \simeq 100$ MeV, which agrees exactly with the quoted result of the detailed study in Ref. [67].

To translate these into an estimate for δE_\pm in our scenario, we need to take into account several modifications. (i) The endpoint locations have changed significantly because the slepton and neutralino masses are different. Therefore also the experimental energy resolution at the endpoint locations is different. (ii) The number of events for slepton pair production is different in our scenario due to the different masses, center-of-mass energy, and luminosity, so that the statistical error on the endpoint determination is different. Determining the effect of (i) requires choosing a parametrization of the detector's energy resolution, which we discuss in the next section and provide in Eqs. (14) and (15). Taking these two factors into account, we can estimate the ratio of our endpoint

energy uncertainty to Martyn's in Ref. [67]

$$\frac{\delta E_{\pm}^{\text{us}}}{\delta E_{\pm}^{\text{Martyn}}} \simeq \frac{\delta E_{\text{exp}}(E = E_{\pm}^{\text{us}})}{\delta E_{\text{exp}}(E = E_{\pm}^{\text{Martyn}})} \times \sqrt{\frac{N_{\text{events}}^{\text{Martyn}}}{N_{\text{events}}^{\text{us}}}}, \quad (13)$$

where we have estimated that the uncertainty in the end-point determination drops with the square root of the number of observed events. Plugging the relevant numbers into the above expression for E_+ , which dominates the error, in fact yields $\frac{\delta E_+^{\text{us}}}{\delta E_+^{\text{Martyn}}} \simeq 1.3$, since the increased number of events in our scenario is partially canceled by the reduced detector resolution at the higher value of E_+ .

Referring again to Eq. (12), we can then estimate that $\delta M_{\chi_1^0} \simeq 1.4$ GeV for our scenario. In other words, for a neutralino mass of 2 GeV, the mass can be determined to about 70% accuracy. This suggests that a useful mass measurement can be performed for very light neutralinos, and in particular in the range $M_{\chi_1^0} \sim 5$ GeV that is particularly interesting for dark matter phenomenology, sub-GeV accuracy should be possible.

On the other hand, if we carry out the same estimate for a 2 GeV neutralino and instead a 100 GeV \tilde{e}_R , we find that the factor in front of the square root in Eq. (12) is now 40, and the cross section for selectron pair production is also lower so that the statistical uncertainty is larger. In this case we find $\delta M_{\chi_1^0} \simeq 15$ GeV, suggesting that in this case at best an upper limit on the neutralino mass can be set.

While this simple estimate gives a qualitative illustration of the difficulty of measuring a light neutralino mass, we would would like to check it with a more thorough analysis and more precisely quantify the accuracy possible for a light neutralino mass measurement at the ILC. We do this in the next section.

IV. SIMULATION OF NEUTRALINO MASS MEASUREMENT FROM SLEPTON PAIR PRODUCTION

Thanks to the simple kinematics of slepton pair production, it is possible to estimate the precision for a χ_1^0 mass measurement at the ILC from a rather simple Monte Carlo simulation. We describe this in the following.

First, the number of produced slepton pairs for a given centre-of-mass energy \sqrt{s} and luminosity \mathcal{L} is calculated for a beam polarisation of $(\mathcal{P}_{e^-}, \mathcal{P}_{e^+}) = (+80\%, -60\%)$ using the program **SPheno** [72], which implements the cross section formulae from Refs. [73–76]. This choice of signs for the beam polarization maximizes the production cross-section. For each event, two lepton energies are thrown according to a flat probability density distribution between E_- and E_+ . In order to take effects caused by beamstrahlung into account, E_- and E_+ are evaluated for each event using the reduced centre-of-mass energy $\sqrt{s'}$ which is thrown according to the luminosity

spectrum computed by GUINEA PIG [77]. The energy difference $\sqrt{s} - \sqrt{s'}$ is lost in the form of beamstrahlung photons. As a result the sharp edges in the lepton energy spectrum are smoothed out a bit.

The resulting lepton energies are subsequently smeared according to the expected momentum and energy resolution. This smoothes out the edges even further. For electrons, the minimum of track momentum resolution and the energy resolution of the electromagnetic calorimeter (ECAL) for the considered electron energy is employed. In the case of muons, the momentum resolution of the tracking system is always used. For these quantities, the following parametrizations are used:

$$\Delta \frac{1}{p_T} = 1 \cdot 10^{-4} \text{ GeV}^{-1} \quad (\text{tracker}), \quad (14)$$

$$\frac{\Delta E}{E} = \frac{0.166}{\sqrt{E/\text{GeV}}} \oplus 0.011 \quad (\text{ECAL}). \quad (15)$$

Any polar angle dependence of the tracker resolution is neglected. Instead a rather conservative average resolution is applied (compare e.g. Ref. [78]). We checked that the results do not depend strongly on the assumed tracker resolution since for the considered SUSY masses, the χ_1^0 mass measurement is dominated by the calorimeter resolution. The above parametrization of the ECAL resolution which we employ is the one obtained with a detector prototype in test beam measurements [79]. The effects of a limited detector acceptance, signal selection cuts and inefficiencies in the electron and muon reconstruction are approximately accounted for by applying an overall efficiency of 50%. This roughly corresponds to the values obtained in Ref. [67] using a more detailed simulation. This more detailed study also showed that background rates are rather small [70]. Therefore, outside of this overall efficiency, we neglect backgrounds completely in our study.

The edge positions of the lepton spectrum obtained in the described way are finally fitted using an unbinned likelihood fit. The fitted shapes are

$$f_-(E) = \begin{cases} \frac{1}{2} \left[\text{erf} \left(\frac{E - \hat{E}_-}{\sqrt{2}\sigma_1^-} \right) + 1 \right] & : E < \hat{E}_- \\ \frac{1}{2} \left[\text{erf} \left(\frac{E - \hat{E}_-}{\sqrt{2}\sigma_2^-} \right) + 1 \right] & : E \geq \hat{E}_- \end{cases} \quad (16)$$

for E_- and

$$f_+(E) = \begin{cases} \frac{1}{2} \text{erfc} \left(\frac{E - \hat{E}_+}{\sqrt{2}\sigma_1^+} \right) & : E < \hat{E}_+ \\ \frac{1}{2} \text{erfc} \left(\frac{E - \hat{E}_+}{\sqrt{2}\sigma_2^+} \right) & : E \geq \hat{E}_+ \end{cases} \quad (17)$$

for E_+ . If one chooses $\sigma_1^\pm = \sigma_2^\pm$, Eqs (16) and (17) are the results of a convolution of an upward and a downward step function with a Gaussian. Between the nominal edge positions E_- and E_+ the shape of the lepton energy spectrum is influenced by beamstrahlung and energy/momentum resolution, whereas outside the nominal edge positions, the shape is only determined by the energy/momentum resolution. For this reason σ_1^\pm and σ_2^\pm

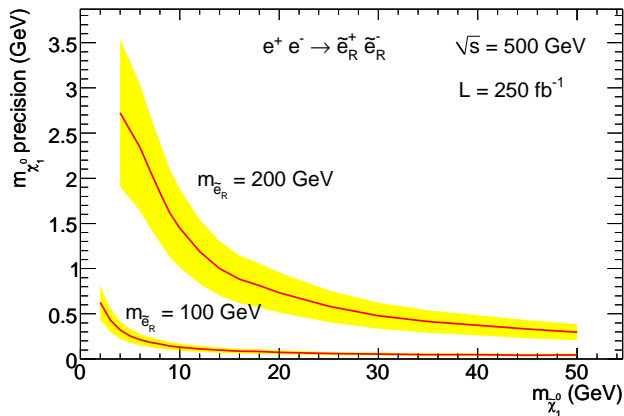


FIG. 1: Estimated precision of the χ_1^0 mass measurement from $\tilde{e}_R \tilde{e}_R$ production as function of the χ_1^0 mass for \tilde{e}_R masses of 100 GeV and 200 GeV. The yellow bands represent the estimated uncertainty of 30% related to the simplifications of the Monte Carlo simulation used. The assumed centre-of-mass energy is $\sqrt{s} = 500$ GeV, the integrated luminosity $\mathcal{L} = 250 \text{ fb}^{-1}$ and the beam polarization $(\mathcal{P}_{e^-}, \mathcal{P}_{e^+}) = (+80\%, -60\%)$.

are treated as separate parameters in the fit. The fitted values of the parameters \hat{E}_- and \hat{E}_+ do not in general coincide with the values of E_- and E_+ . The reason is that the asymmetric shape of the beamstrahlung energy spectrum leads to a certain offset. To correct for this bias, a Monte Carlo based calibration procedure is used.

The uncertainty on the edge positions, and thus the masses, is determined by creating toy Monte Carlo datasets. For each toy data set, the fitted and corrected values of E_+ and E_- can be converted into the squared neutralino mass using Eq. (8). The distribution of $m_{\chi_1^0}^2$ from the ensemble toy data sets is approximately Gaussian, and we use the width of this distribution to determine the uncertainty on the mass measurement. For low neutralino masses, the distribution can have support in the unphysical region where $m_{\chi_1^0}^2 < 0$. To account for this, we used the Feldman-Cousins method [80] to have a smooth transition between a mass measurement, which is possible for heavier neutralinos, and an upper bound, which is necessary for very light neutralinos.

Using the described procedure, we agree within roughly 30% with the results in Refs. [66, 67, 70], which were obtained for $M_{\chi_1^0} = 71.9, 96$, and 135 GeV, respectively. Therefore we assign a systematic uncertainty of 30% to our results due to the simplifications of our simulation.

Our estimate of the precision of the χ_1^0 mass measurement from $\tilde{e}_R \tilde{e}_R$ production at the ILC is shown in Fig. 1 as a function of the χ_1^0 mass. The assumed luminosity is 250 fb^{-1} at a centre-of-mass energy of $\sqrt{s} = 500$ GeV with a beam polarization of $(\mathcal{P}_{e^-}, \mathcal{P}_{e^+}) = (+80\%, -60\%)$. Even for χ_1^0 masses as small as 2 GeV, a precision on the χ_1^0 mass measurement of ≈ 0.6 GeV can

be achieved for $M_{\tilde{e}_R} = 100$ GeV.

We find that below about 2 (4) GeV for a 100 (200) GeV selectron, a mass measurement is no longer possible and we can only set an upper bound on the neutralino mass. For example, for $m_{\chi_1^0} = 1$ GeV, the 95 % CL upper limits are 2.5 GeV (7.6 GeV) for a selectron mass of 100 GeV (200 GeV).

We note that the precision of the mass measurement that we obtain in this simulation is roughly a factor of two better than the rough estimate of the precision made in Section III. This is due to the simplistic scaling assumptions made there, namely that the endpoint energy determination accuracies scale like $1/\sqrt{N}$ as the number of events changes, and linearly with the detector resolution as the endpoint energy changes. Using dedicated simulations we find some deviation from this simple scaling which is due in a large part to the effect of beamstrahlung. This more realistic scaling can account for the discrepancy between our estimate and simulation results.

Combining the results from $\tilde{\mu}_R \tilde{\mu}_R$ production with those from $\tilde{e}_R \tilde{e}_R$ production does not lead to a sizable improvement of the obtained precision. The reason is the significantly higher cross-section for $\tilde{e}_R \tilde{e}_R$ production due to the additional t -channel contribution, which is especially important for low neutralino masses, leading to a factor of 2 to 3 weaker constraints from $\tilde{\mu}_R \tilde{\mu}_R$ production.

V. SUMMARY AND CONCLUSION

A light neutralino in the several-GeV mass range is currently of special phenomenological interest. Recent dark matter direct detection experiments hint at the possible existence of such a light particle. On the other hand, recent phenomenological analyses claim that an MSSM light neutralino dark matter candidate has a lower bound on its mass around 7 GeV.

If a light neutralino exists, it would therefore be extremely important to obtain an accurate determination of its mass. Techniques for measuring neutralino masses at the ILC have been developed and shown to have extraordinary precision for the more conventional 50–100 GeV range. These techniques, however, have not been studied for much lighter neutralinos.

In this paper, we have studied one of these techniques—measuring the lepton energy spectrum in slepton pair production events—and determined its usefulness for the measurement of very light neutralino masses. We showed with a simulation that this technique continues to have useful accuracy for a neutralino with a mass as low as a few GeV. For example, we showed that it is possible to measure the mass of even a 2 GeV neutralino to sub-GeV accuracy if the mass of the right-handed selectron is 100 GeV. For a 200 GeV selectron, the precision is about 2.7 GeV for a 4 GeV neutralino. For even lighter neutralinos, we showed that this method

can give an 95 % CL upper bound of 2.5 (7.6) GeV for a 100 (200) GeV selectron.

Such mass measurements at the ILC will thus be indispensable in testing the MSSM thermal cold dark matter picture if a very light neutralino exists.

Appendix A: Chargino and Neutralino Mixing

Here we summarize the mixing of the electroweak gauginos and Higgsinos, which we use in the paper. The spin-1/2 superpartners of the W^\pm gauge bosons and the scalar charged Higgs field, H^\pm mix after electroweak symmetry breaking. The resulting mixing matrix in the wino, Higgsino basis is given by [81]

$$\begin{pmatrix} M_2 & \sqrt{2}M_W \sin \beta \\ \sqrt{2}M_W \cos \beta & \mu \end{pmatrix}. \quad (\text{A1})$$

Here M_2 is the SU(2) soft breaking gaugino mass. μ is the supersymmetric Higgs mixing parameter, $\tan \beta$ is the ratio of the vacuum expectation values of the two Higgs doublets and M_W is the mass of the W boson.

Similarly the spin-1/2 superpartners of the W^0 and B gauge bosons as well as of the two CP-even neutral

Higgs mix after electroweak symmetry breaking. The 4×4 mixing matrix is given in the bino, wino, Higgsino basis by [82]

$$\begin{pmatrix} M_1 & 0 & -M_Z s_w c_\beta & M_Z s_w s_\beta \\ 0 & M_2 & M_Z c_w \cos \beta & -M_Z c_w s_\beta \\ -M_Z s_w c_\beta & M_Z c_w \cos \beta & 0 & -\mu \\ M_Z s_w s_\beta & -M_Z c_w s_\beta & -\mu & 0 \end{pmatrix}. \quad (\text{A2})$$

Here M_1 denotes the supersymmetry breaking bino mass. Furthermore $s_w \equiv \sin \theta_w$, $c_w \equiv \cos \theta_w$ and θ_w is the electroweak mixing angle. M_Z denotes the Z boson mass.

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